Math 1320: Graphing Logarithmic Functions

How do I graph a logarithmic function? Like any other graph that is plotted by hand, we want to start with a table of coordinates ($x-$ and $y-$ values). In order to capture the behavior of the graph, the more points we plot the better. I recommend plotting at least 6 points.

How do I evaluate logarithms?
Consider the three logarithmic equations below:

$f(x) = \log(x-1)$ $\qquad$ $g(x) = \ln(2x)$ $\qquad$ $h(x) = \log_3 x$

x	$f(x) = log\,(x-1)$
1	$log\,(1-1) =?$
2	$log\,(2-1) =?$
3	$log\,(3-1) =?$
4	$log\,(4-1) =?$
5	$log\,(5-1) =?$
6	$log\,(6-1) =?$

x	$g(x) = ln\,(2x)$
1	$ln\,(2\cdot 1) =?$
2	$ln\,(2\cdot 2) =?$
3	$ln\,(2\cdot 3) =?$
4	$ln\,(2\cdot 4) =?$
5	$ln\,(2\cdot 5) =?$
6	$ln\,(2\cdot 6) =?$

x	$h(x) = log_3(x)$
1	$log_3(1) =?$
2	$log_3(2) =?$
3	$log_3(3) =?$
4	$log_3(4) =?$
5	$log_3(5) =?$
6	$log_3(6) =?$

After plugging in the respective values for x, all that's left is to evaluate the logarithm. But how? Well, for $f(x)$ and $g(x)$, we may use the function buttons on our scientific calculators:

- For $f(x)$, when $x = 1$, in our calculator we use the following button sequence:
  LOG (1 - 1) ENTER
- For $g(x)$, when $x = 1$, in our calculator we use the following button sequence:
  LN (2 · 1) ENTER
- Continue the process for each value of x.

But we don't have a calculator button to evaluate a logarithm of base 3. We need a special property of logarithms to graph $h(x)$:

The Change-of-Base Property	
Common Logarithms	Natural Logarithms
$log_b M = \frac{log\,M}{log\,b}$	$log_b M = \frac{ln\,M}{ln\,b}$

Applying the change-of-base property, we can use a scientific calculator to evaluate $h(x)$ at each value of x.

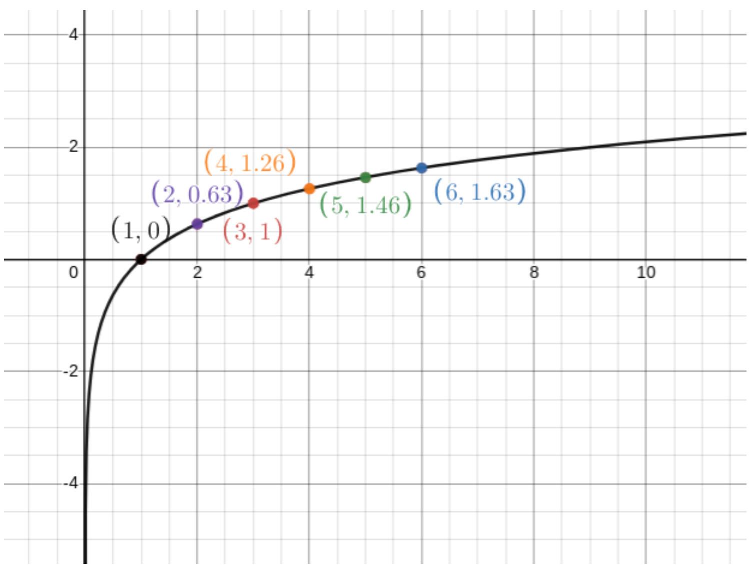
Example 1. Graph the equation $h(x) = \log_3 x$.

I copied the table from above and added an extra column for applying the change-of-base property:

x	$h(x) = log_3(x)$	**Apply Change-of-Base Property**	**Coordinates**
1	$log_3(1) = ?$	$log_3(1) = \frac{log\ 1}{log\ 3} = \frac{ln\ 1}{ln\ 3} = 0$	(1, 0)
2	$log_3(2) = ?$	$log_3(2) = \frac{ln\ 2}{ln\ 3} \approx 0.63$	(2, 0.63)
3	$log_3(3) = ?$	$log_3(3) = \frac{ln\ 3}{ln\ 3} = 1$	(3, 1)
4	$log_3(4) = ?$	$log_3(4) = \frac{ln\ 4}{ln\ 3} \approx 1.26$	(4, 1.26)
5	$log_3(5) = ?$	$log_3(5) = \frac{ln\ 5}{ln\ 3} \approx 1.46$	(5, 1.46)
6	$log_3(6) = ?$	$log_3(6) = \frac{ln\ 6}{ln\ 3} \approx 1.63$	(6, 1.63)

As you can see in the first row, it doesn't matter if you change the base to be in terms of the common log or the natural log. They are equivalent. You may choose whichever form you prefer.

Now, let's plot the six coordinates on a coordinate plane and sketch the curve:



⋆ Note: We know that the graph will approach $x = 0$, but never touch or cross the line, as $\log_3 0 = \frac{\log 0}{\log 3}$ is undefined, since $\log 0$ is undefined (there is no exponent that satisfies $10^? = 0$). Therefore, $x = 0$ is a vertical asymptote of the graph.

Practice applying the change-of-base property to evaluate the logarithms. Use a scientific calculator and round your answer to two decimal places.

1. $\log_5 7$ [≈ 1.21]
2. $\log_9 4$ [≈ 0.63]
3. $\log_1 8$ [undefined]
4. $\log_{12} 1$ [0]